Calibration and Simulation of an Automated Vehicles Highway Traffic*

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Abstract

In this paper, we show the feasibility of the traffic of automated vehicles on a single lane highway. We first determine the parameters of vehicles, the local control of the cars and the sizing of ramps, then we make the simulation of a ring.

1 Introduction

The purpose of this paper is to show the feasibility of the traffic of automated vehicles on a single lane highway and determine the capacity it is possible to obtain. We will consider the case when inter-vehicle spacings are computed to avoid collisions in case of emergency stop.

In the first section, we determine the parameters of vehicles and the local control of the cars. We then examine the sizing of ramps to get compatible inputs and outputs with the main traffic. See [1] for a similar work for platooning.

The second section is devoted to the simulation of a ring, adapting the sizing of input and output ramps and examining the influence of input-output on the main traffic.

Similar modeling and studies can be found in [2].

All the simulations and computations have been made with the Scientific Software Scilab [3].

2 Calibration of vehicle and traffic parameters

In this section, we first determine the influence of the parameters of the vehicles (speed and braking) on the mean capacity of a section. This will allow us to deduce validity domains for the parameters. Then, using these values, we will compute the characteristics of the input and output ramps. Note that there is no control on the traffic.

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2.1 Modeling

We consider a sequence of vehicles: $(i+1)\cdots i\cdots 2 \quad 1$. Vehicle $i+1$ follows vehicle i. The characteristics of vehicle i are: length lg_i ranging from $lg_{\min}$ to $lg_{\max}$, braking capability γ_i ranging from $\gamma_{\min}$ to $\gamma_{\max}$, speed V_i.

The spacing between vehicles $i+1$ and i is ds_i.

The reaction delay of vehicle $i+1$ with respect to vehicle i is tr_{i+1}.

We suppose that at time $t=0$, the front of vehicle $i+1$ is at $x=0$ and that the back of vehicle i is at $x=ds_i$.

Then it is easy to write down the equations of motion of the front of vehicle $i+1$:

- between $t=0$ and $t=tr_{i+1}$, vehicle $i+1$ goes with constant speed V_{i+1};
- after time $t=tr_{i+1}$, vehicle $i+1$ decelerates linearly, so its speed is $-\gamma_{i+1}(t-tr_{i+1})+V_{i+1}$, and it stops at time $ta_{i+1}=tr_{i+1}+V_{i+1}/\gamma_{i+1}$.

To get the spacing $x_{i+1}(t)$, we need to integrate the speed and we obtain:

$$x_{i+1}(t)=\begin{cases} V_{i+1}t \quad \text{when} \quad 0\le t\le tr_{i+1} \\ -\dfrac{1}{2}\gamma_{i+1}t^2+(\gamma_{i+1}tr_{i+1}+V_{i+1})t-\dfrac{1}{2}\gamma_{i+1}tr_{i+1}^2 \\ \qquad \text{when} \quad tr_{i+1}\le t\le ta_{i+1}\end{cases}$$

The equation of motion of the back of vehicle i is simpler because vehicle i decelerates linearly until the stopping time $ta_i=V_i/\gamma_i$:

$$x_i(t)=ds_i-\frac{1}{2}\gamma_i t^2+V_i t \quad \text{when} \quad 0\le t\le ta_i$$

We want to avoid collisions between vehicles, so we must have at any time t:

$$x_i(t)>x_{i+1}(t) \quad \text{for} \quad 0<t<\max(ta_i,ta_{i+1}) \qquad (1)$$

The value of ds_i satisfying constraint (1) depends on time t and on the values of vehicle characteristics and reaction delay. We have considered all the various cases to compute spacing ds_i.

In the following, we suppose that the reaction delay is constant, equal to *tr*. We also suppose that the speed of the vehicles is constant, equal to V with a measurement error V_ϵ. So we have: $V_i = V(1 - V_\epsilon)$ and $V_{i+1} = V(1 + V_\epsilon)$.

According to the errors on the speed of the vehicles and to the various braking capabilities, it is possible to compute a maximal value $ds_{\max}$ and a minimal value $ds_{\min}$ of the spacing ds_i. The former corresponds to the worse case for the capacity and the latter corresponds to the best case. Then the mean maximal capacity $\bar{C}_{\max}$ of the highway section is:

$$\bar{C}_{\max} = \frac{2\,V}{lg_{\min} + lg_{\max} + ds_{\min} + ds_{\max}} \qquad (2)$$

2.2 Influence of parameters

We study here the influence of the various parameters on the mean maximal capacity of a section. We suppose that we have a uniform repartition of the braking capability: this gives us a mean maximal capacity which depends on the mean lengths of the vehicles and on the mean spacings between the vehicles. The mean spacings between the vehicles are computed so that there is no collision between the vehicles in case of emergency stop. In a few cases we will speak of the *wall security index* which corresponds to the number of broken vehicles when there is a crash against a non moving obstacle.

In the following we suppose that the reaction delay of the vehicles is equal to 0.12 s and that the length of the vehicles ranges from 3 m to 5 m.

2.2.1 Influence of the braking capability

Figure 1 shows the mean maximal capacity as a function of speed for various braking capabilities. It is clear that it is better to have the same braking rate for all the vehicles. For instance a constant braking capability of $8\,\mathrm{m/s}^2$ is better than a non constant braking capability ranging from $8\,\mathrm{m/s}^2$ to $10\,\mathrm{m/s}^2$: in this case the mean maximal capacity decreases from 8485 to 6881 vehicles per hour.

But it is easy to show that the variation of the mean maximal capacity w.r.t. constant braking is not important.

2.2.2 Influence of the error on the speed

Figure 2 shows the variation of the mean maximal capacity as a function of speed for various values of the speed error V_ϵ ($V_\epsilon = 0.01$ corresponds to an error of 1 %), with a braking capability $\gamma = 8\,\mathrm{m/s}^2$. We can see that the influence of the speed error is important.

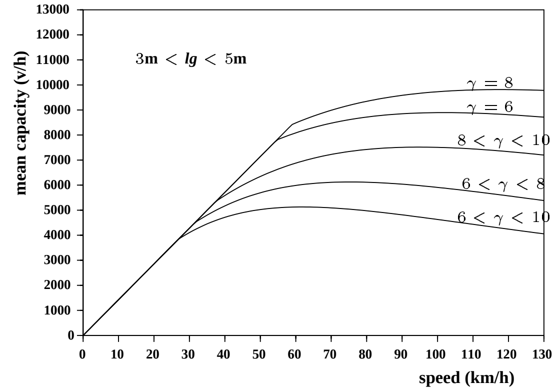

Figure 1: Mean maximal capacity versus speed and braking capability

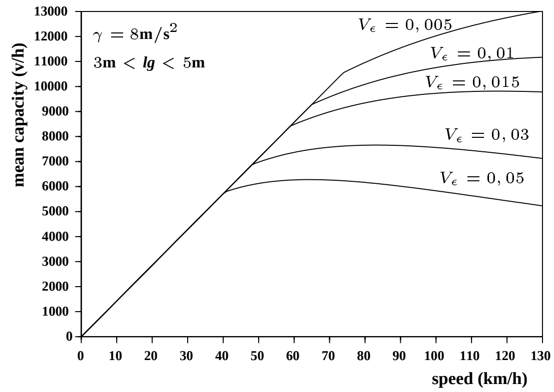

Figure 2: Mean max capacity vs speed and speed error V_ϵ

Moreover, for the standard values $V = 60\,\mathrm{km/h}$, $V_\epsilon = 0.015$ and $\gamma = 8\,\mathrm{m/s}^2$, the minimal security spacing between the vehicles is equal to 3.1 m: this value has the same order of magnitude as the length of the vehicles.

In fact, even a big variation of the braking capability has less influence than the speed error or the variation of the length of the vehicles: indeed with a braking capability $\gamma = 5\,\mathrm{m/s}^2$ and a speed $V = 60\,\mathrm{km/h}$, we get a minimal security spacing equal to 3.7 m and with $\gamma = 12\,\mathrm{m/s}^2$, we get 2.7 m.

2.2.3 Influence of the error on the braking capability

We can do the same for an error γ_ϵ on the braking capability with $V_\epsilon = 0.015$ and $\gamma = 8\,\mathrm{m/s}^2$.

		Error on V			
		0 %	1.5 %	3 %	5 %
Error on γ	0 %	8571	8485	7368	6268
	1.5 %	8571	7902	6924	5943
	3 %	8520	7393	6530	5649
	5 %	7751	6806	6066	5298

The results are summarized on the previous table: variation of the value of the mean maximal capacity versus γ_ϵ and

V_ϵ, with the speed equal to 60 km/h and with $\gamma = 8\,\mathrm{m/s^2}$. This table shows that the speed error has more influence than braking error.

2.2.4 Conclusion

It is clear we have to take constant braking capability, for instance $\gamma = 8\,\mathrm{m/s^2}$ which is an acceptable value for a large variety of vehicles. In the sequel we take $V_\epsilon = 0.015$, but we still have to check if this value is a realistic one because the speed error has a big influence. We take a reaction delay of 0.12 s and the length of vehicles ranging from 3 to 5 meters. With a speed of $V = 60$ km/h, we get a mean maximal capacity of 8484 vehicles per hour.

With all these data, the minimal security inter-vehicle spacing is 3.1 m. If we want more security, we can take an inter-vehicle spacing equal to 6 m, which gives us a very fair mean maximal capacity of 6000 vehicles per hour. Then we have a wall security index equal to 2.

2.3 Sizing on-ramps and off-ramps

The sizes of the ramps must fit the needs of the network flow. With random input-output we will determine the lengths of the ramps and the corresponding traffic on the network section. The parameters are those of the previous paragraphs. The vehicles are equally spaced and have the same braking performance.

In this case the mean maximum capacity is obtained by equation (2) where $ds_{\min} = ds_{\max} = 6$ m.

2.3.1 Poisson entries – Sizing the on-ramp

We suppose that the maximum main flow capacity $C_{\max}$ is 6000 v/h (vehicles per hour). The entries are given by a Poisson process with a mean equal to 1200 v/h. We suppose that the upstream traffic is a Poisson process with mean f. The on-ramp traffic rate is $\lambda = e/3600$, and the arrival rate of free places is $\mu = (C_{\max} - f)/3600$.

Then we get (with $\rho = \lambda/\mu$):

- the mean length of the queue on the on-ramp for a given upstream flow: $\dfrac{\rho/\mu}{1-\rho}$; its variance is $\dfrac{\rho}{(1-\rho)^2}$;
- the maximum upstream flow allowing the entries with a given maximum queue length: $\dfrac{\rho}{1-\rho}$.

Figure 3 shows the queue length on the on-ramp as a function of the upstream flow. We use the curve corresponding to twice the standard deviation for choosing the on-ramp length with the following constraint: the number of waiting vehicles is limited to 20, which gives 80 m for the maximum queue length. The curve shows that the corresponding maximum upstream flow is 4611 v/h with a maximal downstream flow of 5811 v/h, *i.e.* a loss of 189 v/h.

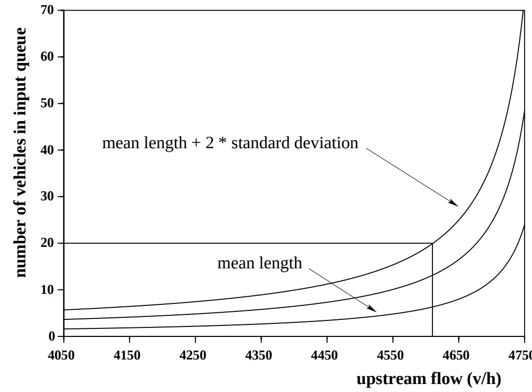

Figure 3: Mean queue length on the on-ramp

Figure 4 shows the mean waiting time on the on-ramp. For the maximal upstream flow (4611 v/h) this time is 16.4s.

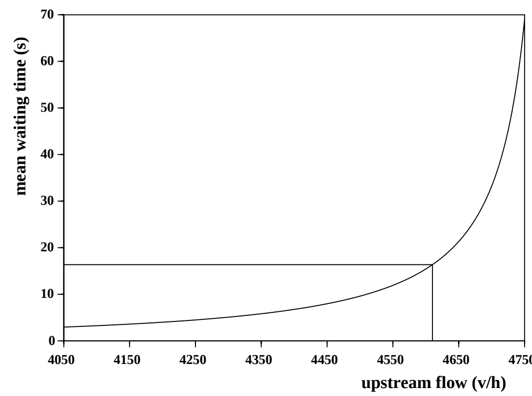

Figure 4: Mean waiting time

We suppose now that the vehicles enter the main flow without time loss and with a constant uniform acceleration of $1.4\,\mathrm{m/s^2}$. This value fits data for small cars.

To increase the speed from 0 to 60 km/h we need 11.9 s and 99 m, which give 180 m for the global on-ramp length, including the 80 m of the queue.

We have to notice that for a traffic of 80 km/h the acceleration phase needs 176 m and so a considerably longer ramp is needed.

2.3.2 Poisson output – Sizing the off-ramp

We perform the same computations for the output: Poisson processes for the output demand (mean: 1200 v/h) and for secondary downstream flow.

We get:

- the length of the output queue for a given output capacity of the secondary flow;

- the minimum value of the output capacity for the satisfaction of the output demand with a given queue length.

Figure 5 shows the mean length of the queue on the off-ramp as a function of the output capacity. With the curve corresponding to twice the standard deviation and with the limitation to 20 vehicles, the need for the output capacity is 1389 v/h.

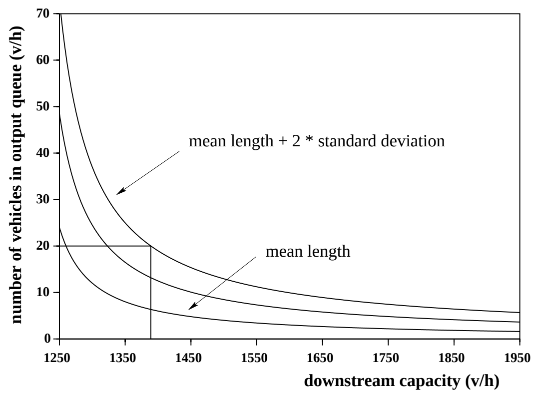

Figure 5: Mean queue length on the off-ramp

We suppose that the braking deceleration is $6\,\mathrm{m/s^2}$; stopping needs 2.8 s which corresponds to 23.1 m. This gives 103 m for the off-ramp length.

If we keep the same length as for on-ramps (180 m) we have 157 m for the queue corresponding to 39 vehicles. In this case the need for the output capacity in only 1295 v/h.

2.3.3 Summary of the parameter values

Calibration

Reaction delay: $0.12\,\mathrm{s}$ Length of vehicles: from $3\,\mathrm{m}$ to $5\,\mathrm{m}$

Braking capability: $8\,\mathrm{m/s^2}$ Speed of vehicles: $60\,\mathrm{km/h}$

Inter-vehicle spacing: $6\,\mathrm{m}$ Maximum mean capacity: $6000\,\mathrm{v/h}$

Input 1200 **v/h**

Upstream flow: $4611\,\mathrm{v/h}$ Queue length: $80\,\mathrm{m}$

Acceleration: $1.4\,\mathrm{m/s^2}$ On-ramp length: $99\,\mathrm{m}$

Output 1200 **v/h**

Output capacity needed: $1295\,\mathrm{v/h}$ Queue length: $157\,\mathrm{m}$

Deceleration: $6\,\mathrm{m/s^2}$ Off-ramp length: $23\,\mathrm{m}$

3 Macroscopic control of input-output

We still consider a macroscopic mean model for a single lane highway ring and we take into account some constraints due to the on-ramps. In particular, we will examine the input control and the calibration of the off-ramps.

3.1 The simplified ring

We consider a ring similar to the Boulevard Périphérique around Paris with the main input-output of the connected highways (see [4, 5]). On the ring, we have 7 input-output, 10 kilometers away each other. Between two on-ramps the ring is split into 10 sections: a section is a piece of 1 km of the ring covered in 1 mn by an automated car at the constant speed of 60 km/h. The capacities of on-ramps and off-ramps and the origin-destination matrix are given.

3.2 Control of entries

The needed strategy must satisfy the capacity constraints of the sections of the ring and of the exit ramps. These constraints are: the limitation to 6000 v/h on the sections and the length of the output queues which may not exceed the length of the ramps.

To clearly understand the different problems, we split the constraints. In a first step we avoid the section capacity constraint to focus on the output ramp limitation. With the previous origin-destination matrix we get such a situation with a common entry rate of 20 v/mn *i.e.* 1200 v/h which is the maximal input flow for one on-ramp.

The simulation process is the following:

- The entries are given by the simulation of a Poisson process with a mean of 20 for a time step of 1 mn (we have the same input on the different on-ramps).
- The output capacities are given. We will consider 3 cases. With infinite capacities we get the maximum possible flow. Then we choose all the capacities equal to the mean of the input (20 v/h) to get the influence of the origin-destination matrix. In the last case, the off-ramp capacities are given by the product of the input demand by the origin-destination matrix: [24, 22, 25, 22, 17, 16, 14].
- The simulation is done with a constant speed for the vehicles. Knowing the whole state of the system (past of the entries), for a new entry demand we can check if the requested output will be available. If not, the entry is refused. This is equivalent to do a real-time modification of the origin-destination matrix.

Figure 6 gives the results without constraints for the off-ramps. The lower part shows the pairs of the output and input flows for the 7 output-input ramps. The higher staircase plot gives the instantaneous traffic on the sections of the boulevard: the mean traffic is 67 v/mn *i.e.* 4020 v/h (this figure corresponds to the traffic after 3 hours).

Figure 7 represents the 7 output flows and the corresponding mean capacities. We clearly see the overloaded off-ramps.

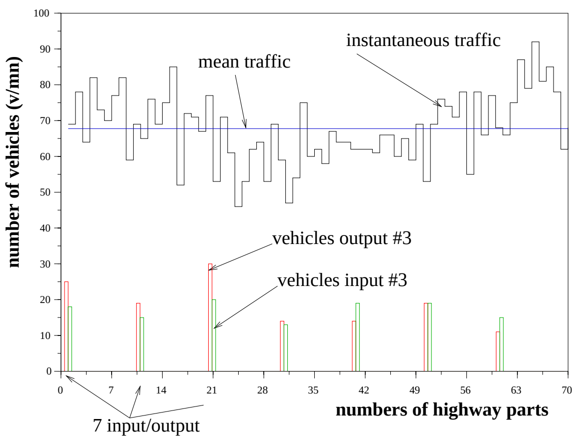

Figure 6: Instantaneous load of the road sections and output-input flows, free behavior

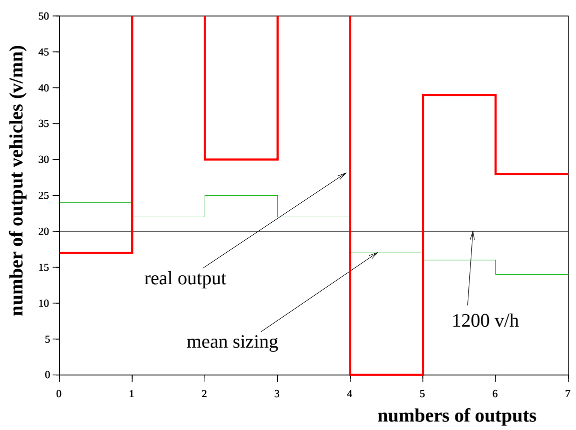

Figure 7: Instantaneous traffic for the different off-ramps, free behavior

We now consider the case when the output capacities are set to 20 v/mn. Figures 8 and 9 are similar to the previous case. The control on the entries in order to meet the off-ramp capacities reduces the mean traffic on the sections to 47 v/mn *i.e.* 2820 v/h.

In the last case we improve the off-ramp capacities: we do a first simulation with a mean calibration and then we observe the saturations and modify the corresponding capacities. Figures 10 and 11 show the results: the traffic is now 53 v/mn *i.e.* 3180 v/h.

These simple examples show the crucial point of the output capacities. From 4020 v/h for the free output, the traffic falls down to 2820 v/h for identical capacities. A simple adjustment leads to a flow of 3180 v/h. In this case we do not take into account the cost necessary for increasing the capacities. This is the topic of the next paragraph.

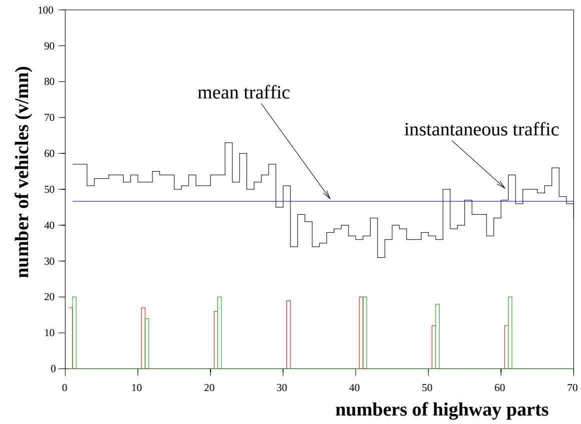

Figure 8: Instantaneous load of the road sections and output-input flows, on-ramp control

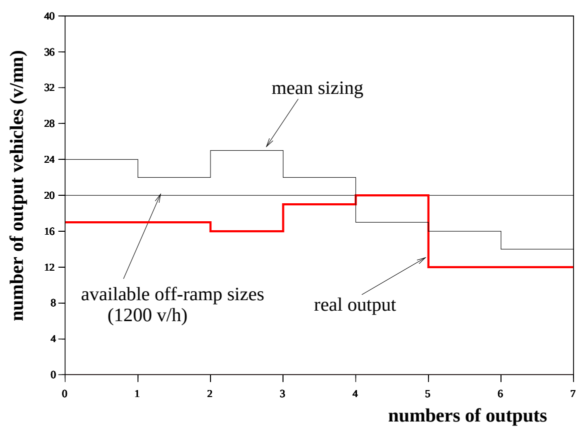

Figure 9: Instantaneous traffic for the different off-ramps, on-ramp control

3.3 Sizing the off-ramps

The problem is to determine the optimal output capacities according to a simple economic cost: the sum of the number of lost kilometers corresponding to the rejected inputs due to the output constraints and the pay off for building the off-ramps. The cost is considered given by the weighted number of the outgoing vehicles.

Let e_i denote the number of entries for the on-ramp i and T_{ij} the origin-destination matrix; d_{ij} is the distance from output-input i to output-input j. For the purpose of sizing the off-ramps we use the origin-destination matrix as a control term: we want to determine u_{ij} (between 0 and 1) which represents the percentage of refused vehicles from i to j. We have:

- number of vehicles outgoing at i: $w_i = \sum_j (1 - u_{ij}) T_{ij} e_j$;
- number of lost kilometers corresponding to the rejected input: $k_i = \sum_j d_{ij} u_{ij} T_{ij} e_j$.

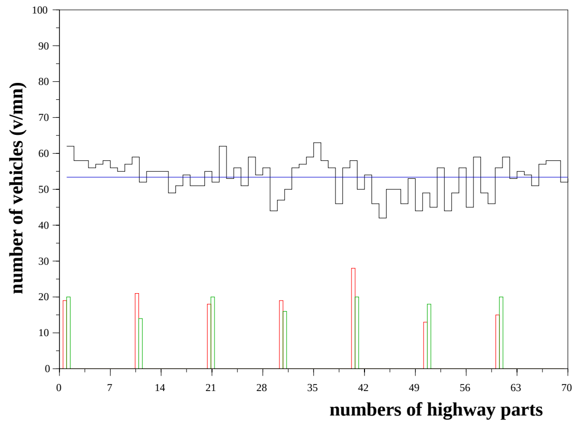

Figure 10: Instantaneous load of the road sections and output-input flows, on-ramp control (adapted off-ramp calibration)

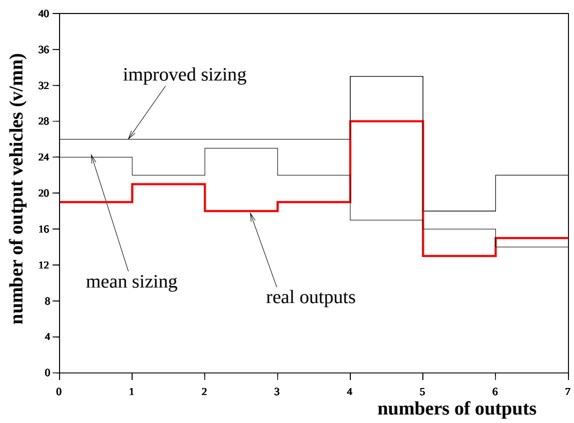

Figure 11: Instantaneous traffic for the different off-ramps, on-ramp control (adapted off-ramp calibration)

The cost function is:

$$J(u) = c_k \sum_j k_i^2 + c_w \sum_j w_i^2$$

where we have 2 weighting coefficients (respectively 1 and 70 in our example).

We start with the previous origin-destination matrix and the same input (20 v/mn for every on-ramp); the mean off-ramp sizes are [24, 22, 25, 22, 17, 16, 14]. We solve the optimization problem with a classical gradient method with constraints. We get the following results:

- rejected entries: [2, 6, 3, 3, 3, 4, 5]
- output values: [20, 18, 20, 18, 14, 13, 11]

The corresponding mean flow is 1980 v/h. This shows the important influence of the constraints (the flow decreases from 3180 to 1980), but note that the costs are not realistic (usually the costs of the output are strongly dependent on the location). These results should be considered only as a measurement of the influence of parameters.

4 Conclusions

We have shown in this paper that using automated vehicles in a highway lane allows to double the traffic of the lane together with improving the security. The characteristics of the vehicles needed for this purpose are easy to have today; the specific equipment of the cars and the highway can be obtained at a low cost. But mixing the automated traffic with the usual one is an important and difficult problem to be studied.

Acknowledgments

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